ON SHOCK WAVES GENERATED BY THE FLOW OF A VISCOUS GAS PAST THIN PROFILES

(OB UDARNYKH VOLNAKH, OBRAZUIUSHCHIKHSIA PRI OBTEKANII TONKIKH PROFILEI VIAZKIM GAZOM)

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Steady supersonic plane flow past thin profiles is considered. It is assumed that the gas is of slight viscosity, and that the shock waves which arise are of small intensity. The flow properties are assumed to be functions, not only of the wave parameter $\tau = x - my$ but, in addition, to depend to a small extent on one of the coordinates. In this case the problem can be reduced to the solution of a quasilinear parabolic equation (2.1).

The resulting equation permits the construction, on an approximate basis, of a complete picture of the behavior of shock waves at any distance from a streamlined profile. The essential influence on the front of the shock wave results from its interaction with a rarefaction wave. The estimated thickness of the front of the shock wave is determined by use of a parabolic equation, showing that as long as the shock front is not coming into contact with the rarefaction wave, its thickness has almost constant value. But as soon as interaction with the wave takes place, there begins a diffusion of the front proportional to the square root of the distance from the profile. In addition, the position of the shock wave changes. Its linearity is violated. In certain cases the solution of the problem posed can be reduced to simplified equations: the quasilinear equation (3.1) and the linear equation (5.3). In the course of the paper the limits of applicability of these equations are shown.

1. The problem of plane steady supersonic flow of a gas past thin pointed profile parallel to the x-axis will be solved. It will be assumed that a shock wave originating at the edge of the profile is of small intensity. Thereby, the characteristic parameters of the flow — velocity $\mathbf{v}$, pressure p, and density of gas ρ, experience disturbances of the same order of smallness in μ. For the sake of convenience the scale of length is taken as the molecular mean free length L. Then the shock wave thickness will be at least of order $1/\mu$ relative to the chosen scale. Therefore, it is natural to assume that, inside the shock wave, differentiation with respect to the coordinates x and y raises the order of smallness of the characteristics of the stream. Due to the dampling of the disturbance, these bounds remain true on the shock wave and also in the refraction wave.

It will be assumed that the dissipation process in the gas is everywhere small. The coefficients of viscosity η, ζ and the coefficient of heat conduction $\varkappa$, in the free stream of gas vary little, in any case μ times slower than the basic characteristic parameters of the flow.

In this case the original system of equations in Eulerian coordinates has the form

$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \eta\triangle\mathbf{v} + (\zeta + {}^1/_3\eta)\nabla(\nabla\mathbf{v}) \tag{1.1}$$

$$\rho T\mathbf{v}\nabla S = \varkappa\nabla^2 T + \sigma_{ik}'\frac{\partial v_i}{\partial x_k} \tag{1.2}$$

$$\nabla(\rho\mathbf{v}) = 0 \tag{1.3}$$

Here S is the entropy, T the absolute temperature and σ_{ik}' the viscous stress tensor,

$$\sigma_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right) + \zeta\delta_{ik}\frac{\partial v_l}{\partial x_l} \tag{1.4}$$

It is necessary to supplement the system of equations (1.1) to (1.3) with the equation of state of the gas and certain relations connecting thermodynamic quantities. In what follows, for the sake of simplicity, the equation of state will be taken as that for an ideal gas $p = \rho RT$.

Processes in the boundary layer are not dealt with, boundary conditions can be written for the outer side of the boundary layer and, at infinity, all disturbances are considered to be damped out.

Firstly, let us transform the heat conduction equation (1.2). From the total differential of the temperature T, expressed as a function of pressure and entropy, it follows that, for steady flow,

$$\nabla T = \left(\frac{\partial T}{\partial p}\right)_S \nabla p + \left(\frac{\partial T}{\partial S}\right)_p \nabla S \approx \left(\frac{\partial T}{\partial p}\right)_S \nabla p \tag{1.5}$$

since, for shock waves of small intensity, the variations of entropy in the interior of the wave are of the second order of smallness in comparison with the increment of pressure. Further

$$\nabla(\nabla T) = \left(\frac{\partial^2 T}{\partial p^2}\right)_S (\nabla p)^2 + \left(\frac{\partial T}{\partial p}\right)_S \nabla^2 p \approx \left(\frac{\partial T}{\partial p}\right)_S \nabla^2 p \tag{1.6}$$

since $(\nabla p)^2 \ll \nabla^2 p$. Thus, to within terms of the third order of smallness inclusive,

$$\rho\mathbf{v}\nabla S \approx \frac{1}{T}\varkappa\left(\frac{\partial T}{\partial p}\right)_S \nabla^2 p \tag{1.7}$$

On the other hand, the total differential of pressure, expressed as a function of volume V and entropy, and the known thermodynamic relations

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{c_p}{T}\left(\frac{\partial T}{\partial p}\right)_S, \qquad \left(\frac{\partial p}{\partial T}\right)_V = -\frac{c_V}{T}\left(\frac{\partial p}{\partial S}\right)_V, \qquad c_p - c_V = T\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_p \tag{1.8}$$

may be similarly expressed as gradients of pressure, density and entropy

$$\nabla p = \left(\frac{\partial p}{\partial \rho}\right)_S \nabla\rho + \left(\frac{T}{\partial T / \partial p}\right)_S \left(\frac{1}{c_V} - \frac{1}{c_p}\right)\nabla S \tag{1.9}$$

Substituting the expression for ∇S in Equation (1.9) from (1.7) the equations of heat flow can be written in the form

$$\mathbf{v}\nabla p = \mathbf{v}\left(\frac{\partial p}{\partial \rho}\right)_S \nabla\rho + \frac{\varkappa}{\rho}\left(\frac{1}{c_V} - \frac{1}{c_p}\right)\nabla^2 p \tag{1.10}$$

If we now substitute ∇p in this from the Navier-Stokes equations then it is possible to exclude one other variable — the pressure p. In addition, within the scope of the assumed accuracy, it can be stated that $(\partial p/\partial \rho)_S$ is equal to the square of the velocity of sound a^2. Moreover, it is easy to show that, for ideal gases,

$$a^2 = a_0^2 - {}^1/_2(\gamma - 1)(v^2 - v_0^2) + O(\mu^2) \qquad (\gamma = c_p / c_V) \tag{1.11}$$

Here a_0 is the velocity of sound in the undisturbed medium, v_0 is the velocity of the undisturbed gas flow.

Terms of the second order of smallness in Equation (1.11) are connected with the dissipation. In Equation (1.10) they do not enter because of the small factor $\nabla\rho$. As a result we obtain the equation in the form

$$\rho(\mathbf{v}\nabla)\mathbf{v}+a^2\nabla\rho=\left[\left(\frac{4}{3}\eta+\zeta\right)+\varkappa\left(\frac{1}{c_V}-\frac{1}{c_p}\right)\right]\triangle\mathbf{v}+$$
$$+\left[\left(\zeta+\frac{1}{3}\eta\right)+\varkappa\left(\frac{1}{c_V}-\frac{1}{c_p}\right)\right]\operatorname{rot}(\operatorname{rot}\mathbf{v}) \qquad (1.12)$$

The curl of the velocity $\mathbf{v}$ in the problem considered has order μ^4. A brief proof of this can be summarized as follows. Let us apply the operator rot to the Navier-Stokes equations (1.1) and express $\nabla\rho$ by means of Equation (1.12), then we obtain an equation of the form

$$\mathbf{v}\nabla\omega+\omega\nabla\mathbf{v}-\frac{\eta}{\rho}\nabla^2\omega=O(\mu^5) \qquad \left(\omega=\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right) \qquad (1.13)$$

Now we can use the method of successive approximation. The second and third terms of the left-hand side of Equation (1.13) have a very high order of smallness, not less than μ^4. Since $v\sim v_0$, then $\nabla\omega$ must be of the same order. On differentiation with respect to the coordinates the order is raised. Therefore, $\nabla^2\omega\sim\mu^5$, $\omega\sim\mu^3$, and the middle term $\omega\nabla v\sim\mu^5$. From this it follows that the derivative of the vorticity must have order μ^5 but the vorticity itself is of the fourth order of smallness.

The result obtained is not unexpected. Although the dissipation processes violate isentropic character of flow, yet, at the same time, they are the causes of smoothing all processes. In consequence of this the flow considered is quasipotential in the approximation assumed.

Thus, in Equation (1.12) the last term on the right-hand side should be dropped.

Then we may state that

$$v_y=\int_{-\infty}^{x}\frac{\partial v_x}{\partial y}dx+\int_{-\infty}^{x}\omega dx+f(y)\approx\int_{-\infty}^{x}\frac{\partial v_x}{\partial y}dx \qquad (1.14)$$

The integral of the velocity has order μ^3. Since $v_y(-\infty,y)\equiv 0$, then $f(y)=0$. A final calculation of the order of quantities in Equation (1.12), the equation of continuity (1.3), Expression (1.11) and Equation (1.14) give the quasilinear equation of the process

$$[m^2+(\gamma+1)M^2u]\frac{\partial u}{\partial x}+2M^2\frac{\partial u}{\partial y}\int_{-\infty}^{x}\frac{\partial u}{\partial y}dx-$$
$$-[1-(\gamma-1)M^2u]\frac{\partial}{\partial y}\left(\int_{-\infty}^{x}\frac{\partial u}{\partial y}dx\right)=\nu M^2\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}\right) \qquad (1.15)$$

The equation is written in dimensionless form, with the coordinates x and y normalized in terms of the mean free path length L, M the Mach number of the undisturbed flow and

$$v_x=v_0(1+u),\qquad v_y=v_0v,\qquad M=v_0/a_0,\qquad m^2=M^2-1$$
$$\nu=\frac{1}{\rho_0v_0L}\left[\left(\frac{4}{3}\eta+\zeta\right)-\varkappa\left(\frac{1}{c_V}-\frac{1}{c_p}\right)\right] \qquad (1.16)$$

Without the right-hand part Equation (1.15) represents the known quasilinear equation of potential flow of an ideal gas. The right-hand part represents dissipation in the medium.

2. In the linear approximation Equation (1.15) is reduced to the one-dimensional wave equation. In the space above the flow past surface ($y>0$) the solution of this equation has the wave form with parameter $\tau=x-my$.

This gives us the right to suppose that the general solution of Equation (1.15) is also a function of the wave parameter τ, but, in addition, it still depends weakly on one of the coordinates x or y. This means that the derivatives of u with respect to this variable are of order μ higher than its derivatives with respect to τ. To be definite it is assumed that $u = u(\tau, \mu y)$.

From this it follows that the velocity u is not constant on the characteristic $\tau = x - my$, and accordingly, in a concrete problem, varies slowly from point to point on the characteristic. It should be noted that analogous methods were studied in problems of wave propagation [1]. Then, by virtue of the statement concerning orders of smallness of derivatives Equation (1.15) takes to form

$$\frac{\partial u}{\partial y} + 2\alpha u \frac{\partial u}{\partial \tau} = \delta \frac{\partial^2 u}{\partial \tau^2} \qquad \left(\alpha = \frac{1}{4}(\gamma + 1)\frac{M^4}{m}, \ \delta = \frac{1}{2}\nu\frac{M^4}{m}\right) \tag{2.1}$$

The equation obtained can be written in another form

$$\frac{\partial \Theta}{\partial y} = \delta \frac{\partial^2 \Theta}{\partial \tau^2} \qquad \left(u = -\frac{\delta}{\alpha}\frac{1}{\Theta}\frac{\partial \Theta}{\partial \tau}\right) \tag{2.2}$$

In a thin boundary layer, the tangential component of velocity varies considerably more quickly than the normal component. Therefore, the boundary condition for Equation (2.1) or (2.2) may be written at the outer side of the boundary layer in the form

$$\mathbf{n}\cdot\mathbf{v} = 0 \qquad (\mathbf{n} \text{ normal to the profile}) \tag{2.3}$$

Thus, the problem may finally be formulated as follows.

Find a bounded function u or Θ $(y \geqslant 0, -\infty < \tau < \infty)$, satisfying Equation (2.1) or (2.2) and the boundary conditions (2.3), prescribed on a streamlined profile. For thin bodies at angle of attack equal to zero, condition (2.3) might be prescribed as $v \approx df/dx$ (for $y = 0$), where $f(x)$ is the profile function. And since $v \approx -mu$, then the boundary conditions for Equations (2.1) and (2.2) respectively have the form

$$u_{y=0} = -\frac{1}{m}\frac{df}{d\tau}\bigg|_{x=\tau}, \qquad \Theta_{y=0} = \exp\left(\frac{\alpha}{m\delta}\frac{df}{d\tau}\right)_{x=\tau} \tag{2.4}$$

Expressions (2.4) are written for the upper half plane.

It should be noted that the quasilinear parabolic equation (2.1) is encountered in aerodynamics. It is known that it can arise in two problems, in the approximate theory of weak unsteady shock waves in real fluid and in the theory of turbulence. However, until now, the derivation of Equation (2.1) was not given, and also the case of steady supersonic flow was not considered.

3. Let us examine the particular case when dissipation processes are infinitesimally small. We then use the quasilinear equation

$$\frac{\partial u}{\partial y} + 2\alpha u \frac{\partial u}{\partial \tau} = 0 \tag{3.1}$$

It is easy to show that its solution may be written in the form

$$\tau = 2\alpha y u + \tau_0(u) \tag{3.2}$$

Here $\tau_0(u)$ is a function defined by condition (2.4) and the first term on the right-hand side exists only in the region of prescribed τ_0.

For a plane wedge with zero angle of attack the profile function is

$$f = \begin{cases} 0 & (x < -l) \\ k(x + l) & (-l < x < 0)\ (k \sim \mu) \\ kl & (x > 0) \end{cases} \tag{3.3}$$

Therefore

$$\tau_0 = \begin{cases} -l & (0 > u > -k/m) \\ 0 & (u < -k/m) \end{cases} \tag{3.4}$$

In Fig.1 tne graph of the profile and the solution u for definite values of y are shown. From physical considerations the left edge of the wave stops at the point B, corresponding to a mean value of the shock perturbation velocity u. In other words, the areas of the triangles ABC and DEC must be equal. The value corresponding to the point D is

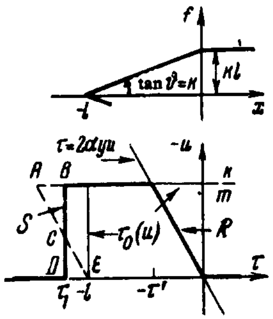

Fig. 1

$$\tau_1 = -l - \alpha k m^{-1} y = -l - {}^1/_2 \tau' \tag{3.5}$$

It defines the angle of inclination of the shock wave S with the direction of the free stream.

$$\tan\varphi = [m - {}^1/_4 (\gamma + 1) M^4 m^{-2} k]^{-1} \tag{3.6}$$

The right slope of the trapezoid-shaped impulse relates to the expansion wave R. With increase in distance y, the thickness of this wave grows and, finally, for the value

$$y_* = \frac{lm}{\alpha k} = \frac{4}{\gamma + 1}\frac{m^2 l}{M^4 k} \tag{3.7}$$

its left edge will stop without fail at the shock wave. At this moment the graph of u degenerates into a triangle (Fig. 2) and the value of the shock velocity begins to fall according to the law

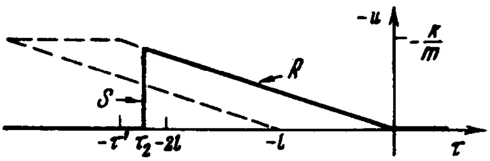

Fig. 2

$$u = -\frac{k}{m}\left(\frac{y_*}{y}\right)^{1/2} \tag{3.8}$$

The position of the shock wave will now be defined by Equation

$$\tau_2^2 = 4lkm^{-1}\alpha y = 2\tau' l \tag{3.9}$$

Thus, if up to the value of $y = y_*$, the inclination of the shock wave is constant, then, for $y > y_*$, with increase in distance from the wedge this slope diminishes and approaches $1/m$, in accordance with

$$\frac{dy}{dx} = \left[m - \left(\frac{lk\alpha}{my}\right)^{1/2}\right]^{-1} \tag{3.10}$$

The physical picture is clear. The interaction of the shock wave S with the expansion wave R inevitably leads to the weakening of the shock wave intensity and, by the same token, to the change of the front position. At large distances from the profile, when the shock wave is essentially damped out, its behavior is determined by a linear hyperbolic equation (see Section 5). Therefore, $\tan\varphi \to 1/m$ (Fig.3). The conclusions reached here coincide exactly with known results [2 and 5 to 7].

To get a complete representation of this interaction it is necessary, even if on an approximate basis, to estimate the thickness of thé shock wave. This can be done only by means of an analysis of the full equation (2.1).

4. According to (2.4), the solution of the heat conduction equation has the form

$$\Theta = \frac{1}{2\sqrt{\pi\delta y}}\int_{-\infty}^{\infty} \exp\left[-\frac{(\tau - \xi)^2}{4\delta y} + \frac{\alpha}{m\delta} f(\xi)\right] d\xi \tag{4.1}$$

For a plane wedge, with relations (3.3)

$$u = -\frac{k}{m}\frac{I}{A+I} \tag{4.2}$$

where

$$I = \frac{1}{\sqrt{\pi}}\int_{\psi_2}^{\psi_1} e^{-x^2}dx \qquad \left(\psi_1 = \frac{\tau+\tau'+l}{b},\ \psi_2 = \frac{\tau+\tau'}{b}\right) \tag{4.3}$$

$$A = \frac{1}{2}\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\psi_3} e^{-x^2}dx\right]\exp\left\{-\frac{\tau'}{b^2}\left[\tau' + 2(\tau + l)\right]\right\} +$$
$$+ \frac{1}{2}\left[1 + \frac{2}{\sqrt{\pi}}\int_0^{\psi_4} e^{-x^2}dx\right]\exp\left[-\frac{\tau'}{b^2}(\tau' + 2\tau)\right] \qquad \left(\psi_3 = \frac{\tau + l}{b},\ \psi_4 = \frac{\tau}{b}\right) \tag{4.4}$$

$$\tau' = \frac{2\alpha k}{m}y = \frac{1}{2}(\gamma + 1)\frac{M^4}{m^2}ky, \qquad b^2 = 4\delta y = 2\frac{M^4}{m}\frac{\nu}{v_0}y \tag{4.5}$$

It is assumed that $|\tau + \tau' + l| \gg b$, $|\tau + \tau'| \gg b$, $|\tau + l| \gg b$, $|\tau| \gg b$. Then, using the asymptotic error integral, we can find an approximate expression for the function u.

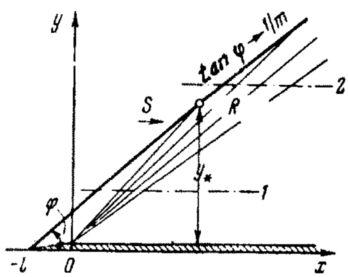

Fig. 3

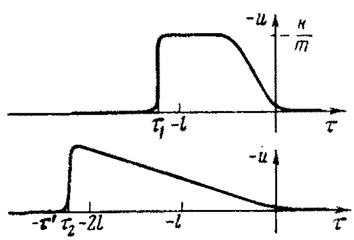

Fig. 4

As in the case of the quasilinear equation (3.1), we must distinguish between two ranges of values of the parameter τ'. When $\tau' < 2l$, u has approximately a trapezoidal form (depending on τ) and, for $|\tau| < \tau'$ it is expressed as follows:

$$u \approx k\tau / m\tau' + O(\mu^2) \tag{4.6}$$

This corresponds to an expansion wave, and, to an accuracy of terms of the second order of smallness, coincides with the solution of (3.1). Small terms in (4.6) indicate a smoothing out of the ends of this wave at the expense of dissipation in the medium (Fig.4).

If $|\tau| > \tau'$, then in the neighborhood $\tau_1 = -l - \tau'/2$

$$u = -\frac{k}{m}\left\{1 + \exp\left[-\frac{2\tau'}{b^2}(\tau + \tau_1)\right]\right\}^{-1} \tag{4.7}$$

Since $|2\tau'(\tau + \tau_1)| \gg b^2$, then the exponent defines the position of the steep shock front, as in (3.5).

Expression (4.7) enables us to estimate the thickness of the front of the shock wave (measured along the direction of the free flow).

$$\Delta\tau = \Delta x \approx \frac{b^2}{\tau'} = \frac{4}{\gamma+1}\frac{\nu m}{v_0 k} \tag{4.8}$$

The expression for the shock front thickness is consistent with formulas previously obtained [5 and 8]. Thus, for example, it is shown [5], that at a small intensity of the shock wave the thickness of the shock is

$$\Delta = \frac{4\nu V^2}{(p_2 - p_1)\,(\partial^2 V / \partial p^2)_S a^2} \tag{4.9}$$

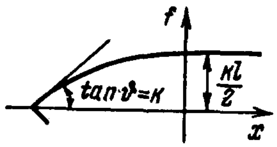

Fig. 5

where p_1 and p_2, respectively, are the values of pressure before and after the shock.

Using the relations

$$\frac{p_2 - p_1}{p_1} \approx \frac{\gamma M^2}{\sqrt{M^2 - 1}}\vartheta, \qquad \left(\frac{\partial^2 V}{\partial p^2}\right)_S = \frac{\gamma+1}{\gamma^2}\frac{V}{p^2}, \qquad \gamma p V = a^2$$

where ϑ is the wedge angle, we can obtain Formula (4.8) from (4.9).

When $\tau' > 2l$, then up to $|\tau| = 2l$, the previous argument is correct, i.e.on this part again $u \approx k\tau/m\tau'$. For $\tau' > |\tau| > l$

$$u \approx \frac{k\tau}{m\tau'}\left[1 - \frac{2\sqrt{\pi}\,\tau}{b}\frac{(\tau+\tau')}{\tau'}\exp\frac{\tau^2 - 2\tau' l}{b^2}\right]^{-1} \tag{4.10}$$

In this expression the exponential term is essential for the formulation of the wave front, the second term gives an expansion wave. In the neighborhood $\tau_2^2 = 2\tau' l$ u is reduced to b times relative to its maximum value. Here τ_2 is the same as in (3.9) and defines the front boundary of the shock wave. An estimate of the thickness of the shock wave front now gives

$$\Delta\tau = \Delta x \approx \frac{b^2}{\sqrt{2\tau' l}} = \frac{2}{\sqrt{\gamma+1}}M^2\frac{\nu}{v_0}\left(\frac{y}{kl}\right)^{1/2} \tag{4.11}$$

Thus, the interaction of the expansion wave with the shock wave causes not only a reduction in the shock wave strength. Together with this there arises a spreading, the extent of viscous interaction is increased.

As is seen from (4.11), the thickness of the shock wave depends, as before, on the parameters of the free flow, the intensity of the wave and the dissipation in the medium. But, in addition, in the case considered, the thickness gradually increases proportionally to the square root of the distance from the streamline profile.

The existence of a diffusive spread of the shock wave occurs in practice for any form of streamlined wedge profile. A calculation, not given here, for example, can be carried out to estimate the thickness of the front for a parabolic profile

$$f(x) = \frac{k}{2l}(l^2 - x^2) \qquad (-l \leqslant x \leqslant 0), \qquad f(x) = \frac{kl}{2} \qquad (x > 0)$$

shown in Fig.5. With the notation used

$$\Delta x \approx \frac{b^2}{\tau'}\left(\frac{\tau'}{l} + 1\right)^{1/2} \tag{4.12}$$

It is clearly seen, that if $\tau'/l \ll 1$ (for $ky/l \ll 1$), then the shock front thickness is again constant. For values $ky/l \gg 1$

$$\Delta x \approx \frac{4}{\sqrt{2(\gamma+1)}}M^2\frac{\nu}{v_0}\left(\frac{y}{kl}\right)^{1/2} \tag{4.13}$$

5. As was seen above, all relations are to be defined in terms of parameters b and τ', which both depend on the distance y. Therefore, as follows from (4.5) there can be two distinct cases (Fig.6).

If $b \gg \tau'$, then Equation (2.1) in practice reduces to a linear form. Indeed, the exponents in expression (4.4) then vary slowly, and therefore

$$I + A \approx 1 + \frac{1}{\sqrt{\pi}}\left(\int_{\psi_2}^{\psi_4} e^{-x^2}dx + \int_{\psi_3}^{\psi_1} e^{-x^2}dx\right) = 1 + z \tag{5.1}$$

where the integral term on the right-hand side, z, is small. Taking this into account we obtain

$$\frac{I}{A+I} \approx I - z = \frac{1}{\sqrt{\pi}}\int_{\psi_4}^{\psi_3} e^{-x^2}dx \tag{5.2}$$

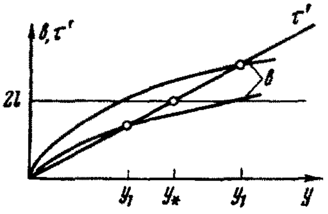

Fig. 6

Now the process is described with sufficient accuracy by

$$\frac{\partial u}{\partial y} = \delta \frac{\partial^2 u}{\partial \tau^2} \tag{5.3}$$

From this it follows that the shock wave and expansion wave decay equally $\Delta\tau \sim b \sim \sqrt{y}$.

For small y the relation $b \gg \tau'$ holds everywhere. But as the coordinate y is increased, the parameter in the inequality referred to necessarily changes sign (for $y = y_1$). The picture of the behavior of the gas in space depends on the distance from the profile to be considered. In case

$$y_1 = \frac{8m^2\nu}{(\gamma+1)^2 M^4 k v_0} < y_* \tag{5.4}$$

on the section $y_1 < y < y_*$ the front of the shock wave has constant thickness, defined only by the stream characteristics. If b and τ' are equal for y_1, exceeding y_*, then the relation giving the thickness of the front in terms of distance remains valid however far we recede from the profile. The interaction of the shock wave with the expansion wave is somewhat intensified by this relation. The last case is possible for very small apex angles of the wedge or for rather dissipative media. In the case $y_1 \gg y_*$ the meaning of the shock wave is assentially lost.

Thus, if $y_1 \ll y_*$, then the problem is described by Equation (2.1). When $y_1 \gg y_*$, sufficient accuracy can be obtained by using the linear equation (5.3).

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EDITORIAL NOTE

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